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ORIGINAL ARTICLE

On Twisted Soft Ideal of Soft Ordered Semigroups

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ABSTRACT

In this paper the concept of twisted extended intersection soft ordered semi group or twisted extended intersection soft partially ordered semi group (briefly, TEISOsG or TEISPOsG), twisted restricted intersection soft ordered semi group or twisted restricted intersection soft partially ordered semi group (TRISOsG or TRISPOsG), twisted extended union soft ordered semi group or twisted extended union soft partially ordered semi group(TEUSOsG or TEUSPOsG), twisted restricted union soft ordered semi group or twisted restricted union soft partially ordered semi group (TRUSOsG or TRUSPOsG), fully twisted extended union soft ordered semi group or fully twisted extended union soft partially ordered semi group (FTEUSOsG or FTEUSPOsG), fully twisted restricted union soft ordered semi group or fully twisted restricted union soft partially ordered semi group (FTRUSOsG or FTRUSPOsG), twisted soft ideal (TS-ideal), restricted twisted soft ideal(RTS-ideal), twisted soft interior ideal (TS-interior ideal), restricted twisted soft interior ideal (RTS-interior ideal), double twisted soft interior ideal (DTS-interior ideal), restricted double twisted soft interior ideal (RDRTS-interior ideal), strong double twisted soft interior ideal (St-DTS-interior ideal), restricted double twisted soft interior ideal (RSt-DRTS-interior ideal), are defined and some interesting results are proved. These concepts are also illustrated with the help of examples.

Keywords: Soft set; Twisted extended intersection (union) soft ordered semigroup; Strong double twisted soft ideal; Strong double twisted interior ideal.

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INTRODUCTION

To face uncertainty, there are many theories like fuzzy sets theory[1], intuitionists fuzzy sets theory[2], interval mathematics theory[3,4] and theory of vague sets[5], theory of probability and rough set theory[6] are presented but these theories have their inherited boundaries due to inefficiency of parameterization tool associated with these theories. No such mathematical theory which can deal uncertainty successfully while occur in environmental areas, engineering and economics. In 1999, Molodtsov7 presented the new approach of soft set theory to deal with uncertainty. This tool solves many problems in different direction and give us proper solutions in various fields like theory of measurement, smoothness of function, theory of operation research, game theory etc. Researchers contributed a lot to extend the concept of soft sets concept in different directions. Aktas and Cagman introduced the basic concepts of soft set theory and compared soft sets to fuzzy and rough sets by providing examples to explain their difference. They also introduced the notation of soft groups[8]. For further development of soft sets, one may refer to[9-13].

The study of ordered structure is discussed on groups and semigroups in classical book of Fuchs[14]. M. I. Ali et al[15] introduced "on lattice ordered soft sets" and explained this concept by using basic operation and examples. They also proved De-Morgan laws. Kehayopulu promoted the concept of ordered semigroup[16]. A semigroup with partially ordered relation is indicated to be ordered semi group (resp. Po-semigroup) if it satisfies the assertion of compatibility for each element of semi group under induced partially ordered relation. Further we can get more information about po-semigroups in[17-22]. Then Jun et al promoted the knowledge of soft ordered semigroups, see[23]. For further information about soft ordered semi group, one may refer to[24-29].

This article is based on 4 sections with section one as introduction. In section two, we discussed some basic concepts. In section three, we defined new concept in soft po-semi group and discussed twisted extended intersection soft ordered semi group and twisted extended union soft ordered semi group and then we applied the concept of restriction on these soft ordered semi group. Further, in section four, we generalized the concept of soft ideals in po-semi group in terms of twisted soft ideal, restricted twisted soft ideals, twisted soft bi-ideals, restricted soft bi-ideal, twisted soft interior ideal, strong double twisted soft interior ideal. The article ends with introduction.

Note that we use abbreviations "Sst", "POsG", L-ideal and R-ideal for soft set, po-semi group, left ideal and right ideal respectively in next work.

PRELIMINARIES

If is non-empty set with binary operation "." and δ satisfies associative and closure properties under " . ". Then $(\delta,.)$ is called semigroup. If T is non-empty subset of δ then T is called sub semigroup of δ if and only if $T^2 \subseteq \delta$. A non-empty subset T of semigroup δ is indicated to be left (resp. right) ideal of δ if and only if $\delta T \subseteq \delta$ (resp. $T\delta \subseteq \delta$). Now T is said to be ideal if it is both left and right ideal. A semigroup δ with an ordered relation $\leq$ on δ is indicated to be a partially ordered semigroup ("POsG") or ordered semigroup if and only if each $(u,p) \in \leq$ satisfies $(tu,tp) \in \leq$ and $(ut,pt) \in \leq$ where $u,p,t \in \delta$ and denoted by $(\delta,.,\leq)$.

Let $\mathfrak{U}$ be an initial universe and $P(\mathfrak{U})$ is power set of $\mathfrak{U}$, E be a set of parameters and $V \subseteq E$. Then Molodtsov[7] defined a soft set (Sst) (Γ,V) over U is such that $\Gamma: V \to P(\mathfrak{U})$. In simple words, a "Sst" over $\mathfrak{U}$ is parameterized family of subsets of the universal set $\mathfrak{U}$. For $\mathfrak{z} \in V$, $\Gamma(\mathfrak{z})$ may be considered as the set of $\mathfrak{z}$-approximations of the "Sst" (Γ,V). Let δ be a semigroup and (Γ,δ) be a "Sst" over U. Then (Γ,δ) is indicated to be an intersection soft set of δ over U if $\Gamma(\mathfrak{z}p) \supseteq \Gamma(\mathfrak{z}) \cap \Gamma(p)$ for all $\mathfrak{z}, p \in \delta$.

2.1 Definition[15]

A "Sst" (F,A) is indicated to be soft twisted subset of $(\mathcal{T},B)$ over a common universe $\mathfrak{U}$ if

(1) $A \subseteq B$.

(2) $\mathcal{T}(t) \subseteq F(t)$ for all $t \in A$.

We write $(F,A) \mathrel{\hat{\subseteq}} (\mathcal{T},B)$.

2.3 Definition [24]

Let δ be a "POsG". Then a "Sst" (Γ,V) over δ is indicated to be a soft partially ordered semi group (SPOsG) if for all $\mathfrak{z} \in V$, $\Gamma(\mathfrak{z}) \in P(\delta)$ is sub semigroup of δ.

2.4 Proposition [24]

Let (F,T) and (Γ,L) be "POsGs" over δ. If $T \cap L = \emptyset$; then any intersection $(F,T) \,\tilde{\cap}\, (\Gamma,L)$ is "POsG" over δ.

2.5 Example

Let $\delta = \{s_1, s_2, s_3, s_4, s_5\}$ be a semigroup with following "." given by

.	s_1	s_2	s_3	s_4	s_5
s_1	s_1	s_4	s_1	s_4	s_4
s_2	s_1	s_2	s_1	s_4	s_4
s_3	s_1	s_4	s_3	s_4	s_5
s_4	s_1	s_4	s_1	s_4	s_4
s_5	s_1	s_4	s_3	s_4	s_5

The relation "$\preccurlyeq$" is defined on S as follows

$$\preccurlyeq := \left\{ \begin{array}{c} (s_1,s_1);(s_1,s_3);(s_1,s_4);(s_1,s_5);(s_2,s_2); \\ (s_2,s_4);\ (s_2,s_5);(s_3,s_3);(s_3,s_5); \\ (s_4,s_4);(s_4,s_5);(s_5,s_5) \end{array} \right\}$$

. Then $(\delta;.;\preccurlyeq)$ is an ordered semigroup. Let (F,A) be soft set over δ, where $A = \{t,r,p\}$ is the set of parameters.

$F(t) = \{s_1, s_2, s_4\},\ F(r) = \{s_3, s_5\}, F(p) = \delta$.

We define another soft set (G,B) over δ such that $B = \{t,r\}$ and $G(t) = \{s_3, s_5\}, G(r) = \{s_1, s_2, s_4\}$. Clearly $A \cap B \neq \emptyset$, (F,B) and (G,B) are "SPOsG" but $F(t) \neq G(t)$ and $F(r) \neq G(r)$. So $(F,B) \,\tilde{\cap}\, (G,B)$ turns out redundant.

2.6 Definition[28]

Let $(\delta,.,\leq)$ be a "POsG". Then "Sst" (Γ,δ) is indicated to be a soft L-ideal (soft R-ideal) over $\mathfrak{A}$ if

$(1)\ \Gamma(\mathfrak{z}p) \supseteq \Gamma(p)\ (\Gamma(\mathfrak{z}p) \supseteq \Gamma(\mathfrak{z}))\ \forall\, \mathfrak{z}, p \in \delta,$

$(2)\ \mathfrak{z} \leq p \Rightarrow \Gamma(\mathfrak{z}) \supseteq \Gamma(p).$

2.7 Definition [28]

Let $(\delta,.,\leq)$ a "POsG". Then a "Sst" (Γ,δ) is indicated to be a soft bi-ideal over $\mathfrak{A}$ if

$(1)\ \Gamma(\mathfrak{z}pk) \supseteq \Gamma(\mathfrak{z}) \cap \Gamma(k)\ \forall\, \mathfrak{z}, p, k \in \delta,$

$(2)\ \mathfrak{z} \leq p \Rightarrow \Gamma(\mathfrak{z}) \supseteq \Gamma(p).$

RESULTS & DISCUSSION

3. Twisted Extended Intersection and Union Soft Ordered Semigroups

In this section, we propose the concept of twisted extended intersection soft partially ordered semi group or twisted extended intersection soft ordered semi group (TEISPOsG or TEISOsG), twisted restricted intersection soft partially ordered semi group or twisted restricted intersection soft ordered semi group(TRISPOsG or TRISOsG), twisted extended union soft partially ordered semi group or twisted extended union soft ordered semi group(TEUSPOsG or TEUSOsG), twisted restricted union soft partially ordered semi group or twisted restricted union soft ordered semi group (TRUSPOsG or TRUSOsG), fully twisted extended union soft partially ordered semi group or fully twisted extended union soft ordered semi group (FTEUSPOsG or FTEUSOsG) , fully twisted restricted union soft partially ordered semi group or fully twisted restricted union soft ordered semi group (FTRUSPOsG or FTRUSPOsG) and prove some basic properties and results by examples.

3.1. Definition

Let $(\delta,.,\Omega)$ be a "POsG". Then a "Sst" (Γ,δ) over $\mathfrak{A}$ is indicated to be "TEISPOsG" over $\mathfrak{A}$ if

$(1)\ \Gamma(\mathfrak{z}) \cap \Gamma(p) \subseteq \Gamma(\mathfrak{z}p)$ for all $\mathfrak{z}, p \in \delta$.

$(2)\ \mathfrak{z}\Omega p \Rightarrow \Gamma(\mathfrak{z}) \subseteq \Gamma(p)$.

3.2. Definition

Let $(\delta,.,\Omega)$ be a "POsG". Then a "Sst" (Γ,δ) over $\mathfrak{A}$ is indicated to be a "TRISPOsG" over $\mathfrak{A}$ if for $\mathfrak{z}, p \in \delta$

$(1)\ \mathfrak{z}\Omega p$ implies that $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$,

(2) Now $\mathfrak{z}\Omega p$ so $\Gamma(\mathfrak{z}) \cap \Gamma(p) \subseteq \Gamma(\mathfrak{z}p)$.

3.3. Example

Consider $\delta = \{\sigma, \alpha, \pi, \mathfrak{w}, \beta\}$ and define "." On δ such that

.	σ	α	π	$\mathfrak{w}$	β
σ	σ	β	π	$\mathfrak{w}$	β
α	σ	α	π	$\mathfrak{w}$	β
π	σ	β	π	$\mathfrak{w}$	β
$\mathfrak{w}$	σ	β	π	$\mathfrak{w}$	β
β	σ	β	π	$\mathfrak{w}$	β

Then $(\delta, .)$ is semi group. We define relation "Ω"on δ as follows:

$\Omega = \left\{ \begin{matrix} (\sigma,\sigma), (\sigma,\mathfrak{w}), (\alpha,\alpha), (\pi,\pi), (\pi,\beta), \\ (\mathfrak{w},\mathfrak{w}), (\beta,\beta) \end{matrix} \right\}$ is a partially order on δ.Then $(\delta, ., \Omega)$ is "OsG".

Let (Γ, δ) is "Sst" over $\mathfrak{A} = \{6,7,8,9,10\}$. Then $\Gamma(\sigma) = \{6,7\}, \Gamma(\alpha) = \{7,8\}, \Gamma(\pi) = \{6,8,9\}$, $\Gamma(\mathfrak{w}) = \{6,7,9,10\}$ and $\Gamma(\beta) = \{6,8,9,10\}$. Then Clearly condition (7) of definition 3.6 satisfied for $(\sigma,\sigma), (\alpha,\alpha), (\pi,\pi), (\mathfrak{w},\mathfrak{w})(\beta,\beta)$. Now we check condition (2) for $(\sigma, \mathfrak{w}), (\pi, \beta)$

As $\sigma\Omega\mathfrak{w}$ so $\Gamma(\sigma) = \{6,7\} \subseteq \{6,7,9,10\} = \Gamma(\mathfrak{w})$

$\Rightarrow \Gamma(\sigma) \subseteq \Gamma(\mathfrak{w})$.

As

$\pi\Omega\beta$ so $\Gamma(\pi) = \{6,8,9\} \subseteq \{6,8,9,10\} = \Gamma(\beta)$

$\Rightarrow \Gamma(\pi) \subseteq \Gamma(\beta)$.

Further

$\Gamma(\sigma) \cap \Gamma(\sigma) = \{6,7\} \cap \{6,7\} = \{6,7\} \subseteq \{6,7\} = \Gamma(\sigma) = \Gamma(\sigma\sigma)$

$\Rightarrow \Gamma(\sigma) \cap \Gamma(\sigma) \subseteq \Gamma(\sigma\sigma)$.

$\Gamma(\alpha) \cap \Gamma(\alpha) = \{7,8\} \cap \{7,8\} = \{7,8\} \subseteq \{7,8\} = \Gamma(\alpha) = \Gamma(\alpha\alpha)$

$\Rightarrow \Gamma(\alpha) \cap \Gamma(\alpha) \subseteq \Gamma(\alpha\alpha)$.

$\Gamma(\pi) \cap \Gamma(\pi) = \{6,8,9\} \cap \{6,8,9\} = \{6,8,9\} \subseteq \{6,8,9\} = \Gamma(\pi) = \Gamma(\pi\pi)$

$\Rightarrow \Gamma(\pi) \cap \Gamma(\pi) \subseteq \Gamma(\pi\pi)$.

$\Gamma(\mathfrak{w}) \cap \Gamma(\mathfrak{w}) = \{6,7,9,10\} \cap \{6,7,9,10\} = \{6,7,9,10\} \subseteq \{6,7,9,10\} = \Gamma(\mathfrak{w}) = \Gamma(\mathfrak{w}\mathfrak{w}) \Rightarrow \Gamma(\mathfrak{w}) \cap \Gamma(\mathfrak{w}) \subseteq \Gamma(\mathfrak{w}\mathfrak{w})$

$\Gamma(\beta) \cap \Gamma(\beta) = \{6,8,9,10\} \cap \{6,8,9,10\} = \{6,8,9,10\} \subseteq \{6,8,9,10\} = \Gamma(\beta) = \Gamma(\beta\beta) \Rightarrow \Gamma(\beta) \cap \Gamma(\beta) \subseteq \Gamma(\beta\beta)$

$\Gamma(\sigma) \cap \Gamma(\mathfrak{w}) = \{6,7\} \cap \{6,7,9,10\} = \{6,7\} \subseteq \{6,7,9,10\} = \Gamma(\mathfrak{w}) = \Gamma(\sigma\mathfrak{w})$

$\Rightarrow \Gamma(\sigma) \cap \Gamma(\mathfrak{w}) \subseteq \Gamma(\sigma\mathfrak{w})$.

$\Gamma(\pi) \cap \Gamma(\beta) = \{6,8,9\} \cap \{6,8,9,10\} = \{6,8,9\} \subseteq \{6,8,9,10\} = \Gamma(\beta) = \Gamma(\pi\beta)$

$\Rightarrow \Gamma(\pi) \cap \Gamma(\beta) \subseteq \Gamma(\pi\beta)$.

$\Rightarrow (\Gamma, \delta)$ is "TRISPOsG".

3.4. Proposition

If $(\delta, ., \Omega)$ be a "POsG" and (Γ, δ) be a "TEISPOsG" over $\mathfrak{A}$. Then prove that (Γ, δ) is "TRISPOsG" over $\mathfrak{A}$.

Proof:

Let $(\delta, ., \Omega)$ be a "POsG" and (Γ, δ) be a "TEISPOsG" over $\mathfrak{A}$. To prove (Γ, δ) "TRISPOsG" over $\mathfrak{A}$. As (Γ, δ) is a "TEISPOsG" over $\mathfrak{A}$ so for each $\mathfrak{z}, p \in \delta$ such that $\Gamma(\mathfrak{z}) \cap \Gamma(p) \subseteq \Gamma(\mathfrak{z}p)$ and $\mathfrak{z}\Omega p$ implies that $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$. Now we can say for each $(\mathfrak{z}, p) \in \Omega$, $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$ and $\mathfrak{z}\Omega p$ so $\Gamma(\mathfrak{z}) \cap \Gamma(p) \subseteq \Gamma(\mathfrak{z}p)$ implies that (Γ, δ) is "TRISPOsG".

3.5. Remark

Converse of Proposition 3.4 may or may not be true.

3.6. Example

Example 3.3 represents "TRISPOsG" but it is not "TEISPOsG" because in Example 3.3 this condition $\Gamma(\mathfrak{z})\cap\Gamma(p) \subseteq \Gamma(\mathfrak{z}p)$ does not hold for $\sigma, \alpha \in \delta$. As $\Gamma(\sigma) = \{6,7\}, \Gamma(\alpha) = \{7,8\}, \Gamma(\beta) = \{6,8,9,10\}$
so
$\Gamma(\sigma)\cap\Gamma(\alpha) = \{6,7\}\cap\{7,8\} = \{7\} \nsubseteq$
$\{6,8,9,10\} = \Gamma(\beta) = \Gamma(\sigma\alpha)$
because $\sigma\alpha = \beta. \Rightarrow \Gamma(\sigma)\cap\Gamma(\alpha) \nsubseteq \Gamma(\sigma\alpha)$

Which implies that converse of Proposition 3.4 may or may not be possible.

3.7. Definition

Let $(\delta, ., \Omega)$ be a "OsG". Then a "Sst" (Γ, δ) over $\mathfrak{A}$ is indicated to be "TEUSOsG" over $\mathfrak{A}$ if it satisfies

$(1)\ \Gamma(\mathfrak{z}) \cup \Gamma(p) \supseteq \Gamma(\mathfrak{z}p)$ for all $\mathfrak{z}, p \in \delta$.

$(2)\ \mathfrak{z}\Omega p$ such that $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$.

3.8. Definition

Let $(\delta, ., \Omega)$ be a "OsG". Then a "Sst" (Γ, δ) over $\mathfrak{A}$ is indicated to be "TRUSOsG" over $\mathfrak{A}$, if for $\mathfrak{z}, p \in \delta$

$(1)\ \mathfrak{z}\Omega p$ such that $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$.

(2) Now $\mathfrak{z}\Omega p$ so $\Gamma(\mathfrak{z}) \cup \Gamma(p) \supseteq \Gamma(\mathfrak{z}p)$.

3.9. Proposition

If $(\delta, ., \Omega)$ be a "POSG" and (Γ, δ) be a "TEUSPOsG" over $\mathfrak{A}$. Then prove that (Γ, δ) is "TRUSPOsG" over $\mathfrak{A}$.

Proof:

Let $(\delta, ., \Omega)$ be a "POsG" and (Γ, δ) be a "TEUSPOsG" over $\mathfrak{A}$. To prove (Γ, δ) is "TRUSPOsG" over $\mathfrak{A}$. As (Γ, δ) is "TEUSPOsG" over $\mathfrak{A}$ so for each $\mathfrak{z}, p \in \delta$ such that $\Gamma(\mathfrak{z}) \cup \Gamma(p) \supseteq \Gamma(\mathfrak{z}p)$ and $\mathfrak{z}\Omega p$ implies that $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$. Now we can say for each $(\mathfrak{z}, p) \in \Omega$, $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$ and $\mathfrak{z}\Omega p$ $\Gamma(\mathfrak{z}) \cup \Gamma(p) \supseteq \Gamma(\mathfrak{z}p)$ implies that (Γ, δ) is "TRUSPOsG".

3.10. Remark

Converse of Proposition 3.9 may or may not be possible.

3.11. Example

Consider $\delta = \{f, \pi, \beta, \sigma\}$ and define "." on δ such that

Then $(\delta, .)$ is a semi group. We define "Ω" on δ as follows:

$\cdot$	f	π	β	σ
f	f	σ	f	σ
π	f	π	f	σ
β	f	σ	β	σ
σ	f	σ	f	σ

$\Omega = \{(f,f), (f,\sigma), (\pi,\pi), (\beta,\beta), (\sigma,\sigma)\}$.

Now we define "Sst" Γ over $\mathfrak{A}$ where $\mathfrak{A} = \{1, 2, 3, 4, 5\}$ such that

$\Gamma(f) = \{1, 2, 3\} = \Gamma(\sigma), \Gamma(\pi) = \{3, 4, 5\}$,
$\Gamma(\beta) = \{3, 5\}$.

Now first we prove that for each $(\mathfrak{z}, p) \in \Omega$ such that $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$.

Clearly for each $(\mathfrak{z}, \mathfrak{z}) \in \Omega$ such that $\Gamma(\mathfrak{z}) \subseteq \Gamma(\mathfrak{z})$ is satisfied, now we check this condition for $(\mathfrak{z}, p) \in \Omega$ such that

For $(f, \sigma) \in \Omega$ such that
$\Gamma(f) = \{1, 2, 3\} \subseteq \{1, 2, 3\} = \Gamma(\sigma)$
$\Rightarrow \Gamma(\sigma) \subseteq \Gamma(f)$.

Further we prove $\Gamma(g\vartheta) \subseteq \Gamma(g) \cup \Gamma(\vartheta)$ for all $g, \vartheta \in \delta$, such that

$\Gamma(\sigma) \cup \Gamma(\sigma) = \{1, 2, 3\} \cup \{1, 2, 3\} \supseteq$
$\{1, 2, 3\} = \Gamma(\sigma) = \Gamma(\sigma.\sigma) \Rightarrow$
$\Gamma(\sigma) \cup \Gamma(\sigma) \supseteq \Gamma(\sigma.\sigma)$.

$\Gamma(f) \cup \Gamma(\sigma) = \{1, 2, 3\} \cup \{1, 2, 3\} =$
$\{1, 2, 3\} \supseteq \{1, 2, 3\} = \Gamma(\sigma) = \Gamma(f.\sigma)$
$\Rightarrow \Gamma(f) \cup \Gamma(\sigma) \supseteq \Gamma(\sigma.f)$.

$\Gamma(f) \cup \Gamma(f) = \{1, 2, 3\} \cup \{1, 2, 3\} =$
$\{1, 2, 3\} \supseteq \{1, 2, 3\} = \Gamma(f) = \Gamma(f.f)$
$\Rightarrow \Gamma(f) \cup \Gamma(f) \supseteq \Gamma(f.f)$.

$\Gamma(\beta) \cup \Gamma(\beta) = \{3, 5\} \cup \{3, 5\} = \{3, 5\} \supseteq$
$\{3, 5\} = \Gamma(\beta) = \Gamma(\beta.\beta) \Rightarrow$
$\Gamma(\beta) \cup \Gamma(\beta) \supseteq \Gamma(\beta.\beta)$.

$\Gamma(\pi) \cup \Gamma(\pi) = \{3,4,5\} \cup \{3,4,5\} \supseteq \{3,4,5\} = \Gamma(\pi) = \Gamma(\pi.\pi)$
$\Rightarrow \Gamma(\pi) \cup \Gamma(\pi) \supseteq \Gamma(\pi.\pi)$.
$\Rightarrow (\Gamma,\delta)$ is "TRUSPOsG" over $\mathfrak{A}$.

We note that each "TRUSPOsG" is not a "TEUSPOsG" because
$\Gamma(\beta) \cup \Gamma(\pi) = \{3,5\} \cup \{3,4,5\} = \{3,4,5\} \not\supseteq \{1,2,3\} = \Gamma(\sigma) = \Gamma(\beta.\pi)$
This means $\Gamma(\beta) \cup \Gamma(\pi) \not\supseteq \Gamma(\beta.\pi)$.

3.12. Definition

Let $(\delta,.,\Omega)$ be a "OSG". Then a "SOSG" (Γ,δ) over δ is indicated to be "FTEUSOsG" over δ if

$(1)\ \Gamma(\mathfrak{z}) \cup \Gamma(p) \supseteq \Gamma(\mathfrak{z}p)$ for all $\mathfrak{z}, p \in \delta$,

$(2)\ \mathfrak{z}\Omega p$ such that $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$.

Where $\Gamma(\mathfrak{z}) \cup \Gamma(p)$ is subsemigroup of δ.

3.13. Definition

Let $(\delta,.,\Omega)$ be a "POSG". Then a "SPOSG" (Γ,δ) is indicated to be "FTRUSPOsG" over δ if for $\mathfrak{z}, p \in \delta$

$(1)\ \mathfrak{z}\Omega p$ such that $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$.

(2) Now $\mathfrak{z}\Omega p$ so $\Gamma(\mathfrak{z}) \cup \Gamma(p) \supseteq \Gamma(\mathfrak{z}p)$.

Where $\Gamma(\mathfrak{z}) \cup \Gamma(p)$ is sub semigroup of δ.

3.14. Proposition

If $(\delta,.,\Omega)$ be a "POSG" and (Γ,δ) be a "FTEUSPOsG" over δ. Then prove that (Γ,δ) is "RTFUSPOsG" over δ.

Proof:

The proof is similar to proof of proposition 3.9.

3.15. Remark

Converse of Proposition 3.14 may or may not be possible.

3.16. Example

In example 3.11, if we define (Γ,δ) over δ in such a way that
$\Gamma(f) = \{f,\sigma\}, \Gamma(\sigma) = \{f,\sigma,\pi\}, \Gamma(\pi) = \{\pi\}$, $\Gamma(\beta) = \{\beta\}$. Then (Γ,δ) over δ is not a "FTEUSPOsG"
because
$\Gamma(\beta) \cup \Gamma(\pi) = \{\beta\} \cup \{\pi\} = \{\pi,\beta\} \not\supseteq \{f,\sigma,\pi\} = \Gamma(\sigma)$
Implies that $\Gamma(\beta) \cup \Gamma(\pi) \not\supseteq \Gamma(\sigma)$.

Hence converse of Proposition 3.20 may or may not be possible.

4. Twisted soft ideal and twisted Soft interior ideal

In this section, we purposed the concept of twisted soft ideal (TS-ideal), restricted twisted soft ideal(RTS-ideal,), twisted soft bi-ideal(TSB-ideal), restricted twisted soft bi-ideal(RTSB-ideal), twisted soft interior ideal (TS-Interior ideal), restricted twisted soft interior ideal (RTS-Interior ideal), double twisted soft interior ideal (DTS-interior ideal), restricted double twisted soft interior ideal (RDTS-Interior ideal), strong double twisted soft interior ideal (St-DTS-Interior ideal), restricted strong double twisted soft interior ideal (RSt-DTS-Interior ideal) and prove some basic properties and results by examples. For further studies, we use abbreviations L-ideal and R-ideal for left ideal and right ideal.

4.1. Definition

Let $(\mathcal{S},.,\partial)$ be a "POsG". Then a "Sst" $(\mathcal{T},\mathcal{S})$ is indicated to be a "TSL-ideal" $(\mathrm{resp.TSR-ideal})$ over $\mathfrak{A}$ if

$(1)\ \mathcal{T}(\mathrm{u}b) \supseteq \mathcal{T}(b)\ (\mathrm{resp.}\mathcal{T}(\mathrm{u}b) \supseteq \mathcal{T}(\mathrm{u}))$ for all $\mathrm{u}, b \in \mathcal{S}$,

$(2)\ \mathrm{u}\partial b$ then $\mathcal{T}(\mathrm{u}) \subseteq \mathcal{T}(b)$.

4.2 Definition

Let $(\mathcal{S},.,\partial)$ be a "POsG". Then a "Sst" $(\mathcal{T},\mathcal{S})$ is indicated to be a "TS-ideal" over $\mathfrak{A}$ if it satisfies the assertion of "TSL-ideal" and "TSL-ideal".

4.3. Definition

Let $(\mathcal{S},.,\partial)$ be a "POsG". Then a "Sst" $(\mathcal{T},\mathcal{S})$ is indicated to be a "RTSL-ideal" (resp. "RTSR-ideal") over $\mathfrak{A}$ if for $\mathrm{u}, b \in \mathcal{S}$

$(1)\ \mathrm{u}\partial b$ then $\mathcal{T}(\mathrm{u}) \subseteq \mathcal{T}(b)$.

(2)Now $\mathrm{u}\partial b$ so $(\mathrm{u}b) \supseteq \mathcal{T}(b)\ (\mathcal{T}(\mathrm{u}b) \supseteq \mathcal{T}(\mathrm{u}))$.

4.4 Definition Let $(\mathcal{S},.,\partial)$ be a "POsG". Then a "Sst" $(\mathcal{T},\mathcal{S})$ is indicated to be a "RTS-ideal" over $\mathfrak{A}$ if it satisfies the conditions of "RTSL-ideal" and "RTSR-ideal".

4.5. Proposition

Let $(\mathcal{S},.,\partial)$ be a "POsG" and $(\mathcal{T},\mathcal{S})$ be a "TSL-ideal" $(\text{resp.}\text{"TSR}-\text{ideal"})$ over $\mathfrak{U}$. Then show that "RTSL-ideal" (resp. "RTSR-ideal") over $\mathfrak{U}$.

Proof

Let $(\mathcal{S},.,\partial)$ be a "POsG" and $(\mathcal{T},\mathcal{S})$ be a "TSL-ideal" $(\text{resp.}\text{"TSR}-\text{ideal"})$ over $\mathfrak{U}$. To prove $(\mathcal{T},\mathcal{S})$ is "RTSL-ideal" $(\text{resp.}\text{"RTSR}-\text{ideal"})$ over $\mathfrak{U}$. As $(\mathcal{T},\mathcal{S})$ is a "TSL-ideal" $(\text{resp.}\text{"TSR}-\text{ideal"})$ over $\mathfrak{U}$ so for each $\mathrm{u}, b \in \mathcal{S}$ such that $\mathcal{T}(b) \subseteq \mathcal{T}(\mathrm{u}b)$ and $\mathrm{u}\partial b$ implies that $\mathcal{T}(\mathrm{u}) \subseteq \mathcal{T}(b)$. Now we can say that for each $(\mathrm{u},b) \in \partial$, $\mathcal{T}(\mathrm{u}) \subseteq \mathcal{T}(b)$. since $\mathrm{u}\partial b$ so $\mathcal{T}(b) \subseteq \mathcal{T}(\mathrm{u}b)$ or $\left(\mathcal{T}(\mathrm{u}b) \supseteq \mathcal{T}(\mathrm{u})\right)$ implies that $(\mathcal{T},\mathcal{S})$ is "RTSL-ideal" $(\text{resp.}\text{"RTSR}-\text{ideal"})$.

4.6. Remark

Converse of Proposition 4.5 may or may not be possible.

4.7. Example

In example 3.3, if we define "SPOsG" over $\mathcal{S}$ in such a way $\mathcal{T}(\sigma) = \{6,8\}$, $\mathcal{T}(\alpha) = \mathfrak{U}$, $\mathcal{T}(\pi) = \{8,10\}$, $\mathcal{T}(\mathrm{W}) = \{6,7,8\}$, $\mathcal{T}(\beta) = \{7,8,9,10\}$. Then $(\mathcal{T},\mathcal{S})$ is "RTSL-ideal" over $\mathcal{S}$. But $(\mathcal{T},\mathcal{S})$ is not "TSL-ideal" over $\mathfrak{U}$ because

$\mathcal{T}(\alpha) = \mathfrak{U} \nsubseteq \{7,8,9,10\} = \mathcal{T}(\beta) = \mathcal{T}(\pi\alpha)$

because $\pi\alpha = \beta$ implies $\mathcal{T}(\lambda) \nsubseteq \mathcal{T}(\pi\alpha)$. So, converse of Proposition 4.5 may or may not be possible.

4.8. Definition

Let $(\mathcal{S},.,\partial)$ be a "POsG". Then a "Sst" $(\mathcal{T},\mathcal{S})$ is indicated to be "TSB-ideal" over $\mathfrak{U}$ if the following conditions are satisfied, if

(1) $\mathcal{T}(\mathrm{u}b\mathcal{w}) \supseteq \mathcal{T}(\mathrm{u}) \cap \mathcal{T}(\mathcal{w})\ \forall\ \mathrm{u}, b, \mathcal{w} \in \mathcal{S}$

(2) $\mathrm{u}\partial b$ then $\mathcal{T}(\mathrm{u}) \subseteq \mathcal{T}(b)$.

4.9. Definition

Let $(\mathcal{S},.,\partial)$ be a "POsG". Then a "Sst" $(\mathcal{T},\mathcal{S})$ is indicated to be "LRTSB-ideal" (RRTSB-ideal) over $\mathfrak{U}$ if the following assertions are satisfied, if for $\mathrm{u}, b, \mathcal{w} \in \mathcal{S}$

(1) $\mathrm{u}\partial b$ then $\mathcal{T}(\mathrm{u}) \subseteq \mathcal{T}(b)$.

(2) Now $\mathrm{u}\partial b$ then $\mathcal{T}(\mathrm{u}b\mathcal{w}) \supseteq \mathcal{T}(\mathrm{u}) \cap \mathcal{T}(\mathcal{w})$ $(\mathcal{T}(\mathcal{w}\mathrm{u}b) \supseteq \mathcal{T}(\mathcal{w}) \cap \mathcal{T}(b))$.

4.10. Proposition

Let $(\mathcal{S},.,\partial)$ be a "POsG" and $(\mathcal{T},\mathcal{S})$ be a "TSB-ideal" over $\mathfrak{U}$. Then prove that $(\mathcal{T},\mathcal{S})$ is "LRTSB-ideal" (RRTSB-ideal) over $\mathfrak{U}$.

Proof

Let $(\mathcal{S},.,\partial)$ be a "POsG" and $(\mathcal{T},\mathcal{S})$ be a "TSB-ideal" over $\mathfrak{U}$. To prove $(\mathcal{T},\mathcal{S})$ is "LRTSB-ideal" (RRTSB-ideal) over $\mathfrak{U}$. As $(\mathcal{T},\mathcal{S})$ is a "TSB-ideal" over $\mathfrak{U}$ so for each $\mathrm{u}, b, \mathcal{w} \in \mathcal{S}$ such that $\mathcal{T}(\mathrm{u}b\mathcal{w}) \supseteq \mathcal{T}(\mathrm{u}) \cap \mathcal{T}(\mathcal{w})$ and $\mathrm{u}\partial b$ implies that $\mathcal{T}(\mathrm{u}) \subseteq \mathcal{T}(b)$. Now we can say that for each $(\mathrm{u},b) \in \partial$, $\mathcal{T}(\mathrm{u}) \subseteq \mathcal{T}(b)$. Since $\mathrm{u}\partial b$ so

$\mathcal{T}(\mathrm{u}b\mathcal{w}) \supseteq \mathcal{T}(\mathrm{u}) \cap \mathcal{T}(\mathcal{w}) (\mathcal{T}(\mathcal{w}\mathrm{u}b) \supseteq \mathcal{T}(\mathcal{w}) \cap \mathcal{T}(b))$

implies that $(\mathcal{T},\mathcal{S})$ is "LRTSB-ideal" (RRTSB-ideal) over $\mathfrak{U}$.

4.11. Remark

Converse of Proposition 4.10 may or may not be possible.

4.12. Example

Consider $\mathcal{S} = \{\lambda, \eta, \mu, \alpha\}$ and define "." on $\mathcal{S}$ such that

.	λ	η	μ	α
λ	λ	α	λ	α
η	λ	η	λ	α
μ	λ	α	μ	α
α	λ	α	λ	α

Then $(\mathcal{S},.)$ is a semi group. We define "∂" on $\mathcal{S}$ as follows:

$\partial = \{(\lambda,\lambda), (\lambda,\alpha), (\eta,\eta), (\mu,\mu), (\alpha,\alpha)\}$.

Now we define "Sst" $\mathcal{T}$ over $\mathfrak{U}$ where $\mathfrak{U} = \{1,2,3,4,5\}$ such that

$\mathcal{T}(\lambda) = \{1,2,3\} = \mathcal{T}(\alpha)$, $\mathcal{T}(\eta) = \{3,4,5\}$, $\mathcal{T}(\mu) = \{3\}$.

Now first we prove that for each $(g, \vartheta) \in \partial$ such that $\mathcal{T}(g) \subseteq \mathcal{T}(\vartheta)$.

Clearly for each $(u,u) \in \partial$ such that $\mathcal{T}(u) \subseteq \mathcal{T}(u)$ is satisfied, now we check this assertion for $(g, \vartheta) \in \partial$ such that

For $(\lambda, \alpha) \in \partial$ such that

$\mathcal{T}(\lambda) = \{1,2,3\} \subseteq \{1,2,3\} = \mathcal{T}(\alpha)$

$\Rightarrow \mathcal{T}(\alpha) \subseteq \mathcal{T}(\lambda)$.

Now we prove $\mathcal{T}(g\vartheta z) \supseteq \mathcal{T}(g) \cap \mathcal{T}(z)$ for some $g, \vartheta, z \in \mathcal{S}$, such that

$\mathcal{T}(\lambda\lambda\lambda) = \mathcal{T}(\lambda\lambda) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{1,2,3\} \Rightarrow \{1,2,3\} \supseteq \{1,2,3\} = \mathcal{T}(\lambda) \cap \mathcal{T}(\lambda) \Rightarrow \mathcal{T}(\lambda\lambda\lambda) \supseteq \mathcal{T}(\lambda) \cap \mathcal{T}(\lambda)$

.

$\mathcal{T}(\lambda\lambda\eta) = \mathcal{T}(\lambda\eta) = \mathcal{T}(\alpha) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{3,4,5\}$

$\Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\lambda) \cap \mathcal{T}(\eta)$

$\Rightarrow \mathcal{T}(\lambda\lambda\eta) \supseteq \mathcal{T}(\lambda) \cap \mathcal{T}(\eta)$.

$\mathcal{T}(\lambda\lambda\mu) = \mathcal{T}(\lambda\mu) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{3\} \Rightarrow$

$\{1,2,3\} \supseteq \{3\} = \mathcal{T}(\lambda) \cap \mathcal{T}(\mu) \Rightarrow \mathcal{T}(\lambda\lambda\mu) \supseteq \mathcal{T}(\lambda) \cap \mathcal{T}(\mu)$

.

$\mathcal{T}(\lambda\lambda\alpha) = \mathcal{T}(\lambda\alpha) = \mathcal{T}(\alpha) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{1,2,3\} \Rightarrow \{1,2,3\} \supseteq \{1,2,3\} = \mathcal{T}(\lambda) \cap \mathcal{T}(\alpha)$

$\Rightarrow \mathcal{T}(\lambda\lambda\alpha) \supseteq \mathcal{T}(\lambda) \cap \mathcal{T}(\alpha)$.

$\mathcal{T}(\lambda\alpha\lambda) = \mathcal{T}(\alpha\lambda) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{1,2,3\} \Rightarrow \{1,2,3\} \supseteq \{1,2,3\} = \mathcal{T}(\lambda) \cap \mathcal{T}(\lambda) \Rightarrow \mathcal{T}(\lambda\alpha\lambda) \supseteq \mathcal{T}(\lambda) \cap \mathcal{T}(\lambda)$.

$\mathcal{T}(\lambda\alpha\eta) = \mathcal{T}(\alpha\eta) = \mathcal{T}(\alpha) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{3,4,5\}$

$\Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\lambda) \cap \mathcal{T}(\eta)$

$\Rightarrow \mathcal{T}(\lambda\alpha\eta) \supseteq \mathcal{T}(\lambda) \cap \mathcal{T}(\eta)$.

$\mathcal{T}(\lambda\alpha\mu) = \mathcal{T}(\alpha\mu) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{3\} \Rightarrow$

$\{1,2,3\} \supseteq \{3\} = \mathcal{T}(\lambda) \cap \mathcal{T}(\mu) \Rightarrow \mathcal{T}(\lambda\alpha\mu) \supseteq \mathcal{T}(\lambda) \cap \mathcal{T}(\mu)$

$\mathcal{T}(\lambda\alpha\alpha) = \mathcal{T}(\alpha\alpha) = \mathcal{T}(\alpha) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{1,2,3\} \Rightarrow \{1,2,3\} \supseteq \{1,2,3\} = \mathcal{T}(\lambda) \cap \mathcal{T}(\alpha)$

$\Rightarrow \mathcal{T}(\lambda\alpha\alpha) \supseteq \mathcal{T}(\lambda) \cap \mathcal{T}(\alpha)$.

$\mathcal{T}(\eta\eta\lambda) = \mathcal{T}(\eta\lambda) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{3,4,5\} \cap \{1,2,3\}$

$\Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\eta) \cap \mathcal{T}(\lambda) \Rightarrow \mathcal{T}(\eta\eta\lambda) \supseteq \mathcal{T}(\eta) \cap \mathcal{T}(\lambda)$

.

$\mathcal{T}(\eta\eta\eta) = \mathcal{T}(\eta\eta) = \mathcal{T}(\eta) = \{3,4,5\} \supseteq \{3,4,5\}$

$\Rightarrow \{3,4,5\} \supseteq \{3,4,5\} = \mathcal{T}(\eta) \cap \mathcal{T}(\eta) \Rightarrow \mathcal{T}(\eta\eta\eta) \supseteq \mathcal{T}(\eta) \cap \mathcal{T}(\eta)$.

$\mathcal{T}(\eta\eta\mu) = \mathcal{T}(\eta\mu) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{3,4,5\} \cap \{3\} \Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\eta) \cap \mathcal{T}(\mu) \Rightarrow \mathcal{T}(\eta\eta\mu) \supseteq \mathcal{T}(\eta) \cap \mathcal{T}(\mu)$

.

$\mathcal{T}(\eta\eta\alpha) = \mathcal{T}(\eta\alpha) = \mathcal{T}(\alpha) = \{1,2,3\} \supseteq \{3,4,5\} \cap \{1,2,3\} \Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\eta) \cap \mathcal{T}(\alpha)$

$\Rightarrow \mathcal{T}(\eta\eta\alpha) \supseteq \mathcal{T}(\eta) \cap \mathcal{T}(\alpha)$.

$\mathcal{T}(\eta\alpha\lambda) = \mathcal{T}(\alpha\lambda) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{3,4,5\} \cap \{1,2,3\} \Rightarrow$

$\{1,2,3\} \supseteq \{3\} = \mathcal{T}(\eta) \cap \mathcal{T}(\lambda) \Rightarrow \mathcal{T}(\eta\alpha\lambda) \supseteq \mathcal{T}(\eta) \cap \mathcal{T}(\lambda)$.

$\mathcal{T}(\mu\mu\lambda) = \mathcal{T}(\mu\lambda) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{3\} \cap \{1,2,3\}$

$\Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\mu) \cap \mathcal{T}(\lambda) \Rightarrow \mathcal{T}(\mu\mu\lambda) \supseteq \mathcal{T}(\mu) \cap \mathcal{T}(\lambda)$.

$\mathcal{T}(\mu\mu\eta) = \mathcal{T}(\mu\eta) = \mathcal{T}(\alpha) = \{1,2,3\} \supseteq \{3\} \cap \{3,4,5\} \Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\mu) \cap \mathcal{T}(\eta)$

$\Rightarrow \mathcal{T}(\mu\mu\eta) \supseteq \mathcal{T}(\mu) \cap \mathcal{T}(\eta)$.

$\mathcal{T}(\mu\mu\mu) = \mathcal{T}(\mu\mu) = \mathcal{T}(\mu) = \{3\} \supseteq \{3\} \cap \{3\} \Rightarrow \{3\} \supseteq \{3\} = \mathcal{T}(\mu) \cap \mathcal{T}(\mu)$

$\Rightarrow \mathcal{T}(\mu\mu\mu) \supseteq \mathcal{T}(\mu) \cap \mathcal{T}(\mu)$.

$\mathcal{T}(\mu\mu\alpha) = \mathcal{T}(\mu\alpha) = \mathcal{T}(\alpha) = \{1,2,3\} \supseteq \{3\} \cap \{1,2,3\} \Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\mu) \cap \mathcal{T}(\alpha)$

$\Rightarrow \mathcal{T}(\mu\mu\alpha) \supseteq \mathcal{T}(\mu) \cap \mathcal{T}(\alpha)$.

$\mathcal{T}(\alpha\alpha\lambda) = \mathcal{T}(\alpha\lambda) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{1,2,3\}$

$\Rightarrow \{1,2,3\} \supseteq \{1,2,3\} = \mathcal{T}(\alpha) \cap \mathcal{T}(\lambda) \Rightarrow \mathcal{T}(\alpha\alpha\lambda) \supseteq \mathcal{T}(\alpha) \cap \mathcal{T}(\lambda)$

.

$\mathcal{T}(\alpha\alpha\eta) = \mathcal{T}(\alpha\eta) = \mathcal{T}(\alpha) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{3,4,5\} \Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\alpha) \cap \mathcal{T}(\eta)$

$\Rightarrow \mathcal{T}(\alpha\alpha\eta) \supseteq \mathcal{T}(\alpha) \cap \mathcal{T}(\eta)$.

$\mathcal{T}(\alpha\alpha\mu) = \mathcal{T}(\alpha\mu) = \mathcal{T}(\lambda) = \{1,2,3\} \supseteq \{1,2,3\} \cap \{3\} \Rightarrow \{1,2,3\} \supseteq \{3\} = \mathcal{T}(\alpha) \cap \mathcal{T}(\mu) \Rightarrow$

$\mathcal{T}(\alpha\alpha\mu) \supseteq \mathcal{T}(\alpha) \cap \mathcal{T}(\mu)$.

$\mathcal{T}(\alpha\alpha\alpha) = \mathcal{T}(\alpha\alpha) = \mathcal{T}(\alpha) = \{1,2,3\} \supseteq \{1,2,3\} \Rightarrow \{1,2,3\} \supseteq \{1,2,3\} = \mathcal{T}(\alpha) \cap \mathcal{T}(\alpha)$

$\Rightarrow \mathcal{T}(\alpha\alpha\alpha) \supseteq \mathcal{T}(\alpha) \cap \mathcal{T}(\alpha)$.

Implies $(\mathcal{T}, \mathcal{S})$ is "LRTSB-ideal" over $\mathfrak{A}$.

The given example shows that "each "LRTSB-ideal" may or may not be a "RTSB-ideal" because $\mathcal{T}(\eta\alpha\eta) = \mathcal{T}(\alpha\eta) = \mathcal{T}(\alpha) = \{1,2,3\} \not\supseteq \{3,4,5\} \cap \{3,4,5\}$

$\Rightarrow \{1,2,3\} \not\supseteq \{3,4,5\} = \mathcal{T}(\eta) \cap \mathcal{T}(\eta) \Rightarrow \mathcal{T}(\eta\alpha\eta) \not\supseteq \mathcal{T}(\eta) \cap \mathcal{T}(\eta)$.

4.13. Definition

Let $(\delta, ., \Omega)$ be a "POsG". Then a "Sst" (Γ, δ) is indicated to be a "TS-Interior ideal" over $\mathfrak{A}$ if

(1) $\Gamma(\mathfrak{z}tp) \supseteq \Gamma(t)$ for all $\mathfrak{z}, t, p \in \delta$,

(2) $\mathfrak{z}\Omega p$ then $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$.

4.14. Definition

Let $(\delta, ., \Omega)$ be a "POsG". Then a "Sst" (Γ, δ) is indicated to be a "RTS-Interior ideal" over $\mathfrak{A}$ if for $\mathfrak{z}, p \in \delta$ such that

(1) $\mathfrak{z}\Omega p$ then $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$.

(2) Now $\mathfrak{z}\Omega p$ then $\Gamma(\mathfrak{z}tp) \supseteq \Gamma(t)$ for all $t \in \delta$.

4.15. Proposition

Let $(\delta, ., \Omega)$ be a "POsG" and (Γ, δ) be a "Sst" over $\mathfrak{A}$. Then show that, each "TS-Interior ideal" is "RTS-Interior ideal".

Proof:

Let $(\delta, ., \Omega)$ be a "POsG" and (Γ, δ) be a "TS-Interior ideal" over $\mathfrak{A}$. To prove (Γ, δ) is "RTS-Interior ideal" over $\mathfrak{A}$. As (Γ, δ) is a "TS-Interior ideal" over $\mathfrak{A}$ so ($\forall$ $t, b, p \in \delta)$ such that $\Gamma(b) \subseteq \Gamma(tbp)$ and if implies that $\Gamma(t) \subseteq \Gamma(p)$. Now we can say that $\forall$ $(t, p) \in \Omega$, $\Gamma(t) \subseteq \Gamma(p)$. Since $\mathfrak{z}\Omega p$ then $\Gamma(b) \subseteq \Gamma(tbp)$ $(\forall\, b \in \delta)$ implies that (Γ, δ) is "RTS-Interior ideal".

4.16. Definition

Let $(\delta, ., \Omega)$ and $(\mathcal{S}, ., \partial)$ be two "POsG". Then a "SPOsG" (Γ, δ) is indicated to be a "DTS-Interior L-ideal" (resp. DTS-Interior R-ideal) over $\mathcal{S}$ if

(1) $\Gamma(\mathfrak{z}tp) \supseteq \Gamma(t)$ $\forall\, \mathfrak{z}, t, p \in \delta$,

(2) $\mathfrak{z}\Omega p$ then $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$, Where $\Gamma(\mathfrak{z})$ is "L-ideal" (R-ideal) of $\Gamma(p)$.

4.17. Definition

A "SPOsG" (Γ, δ) is indicated to be a "DTS-Interior ideal" over $\mathcal{S}$ if it follows both assertions of "DTS-Interior L-ideal" and "DTS-Interior R-ideal".

4.18 Definition

Let $(\delta, ., \Omega)$ and $(\mathcal{S}, ., \partial)$ be two "POsG". Then a "SPOsG" (Γ, δ) is indicated to be a "RDTS-Interior L-ideal" ("RDTS-Interior R-ideal") over $\mathcal{S}$ if for $\mathfrak{z}, p \in \delta$

(1) $\mathfrak{z}\Omega p$ then $\Gamma(\mathfrak{z}) \subseteq \Gamma(p)$, Where $\Gamma(\mathfrak{z})$ is "L-ideal" (R-ideal) of $\Gamma(p)$.

(2) $\Gamma(\mathfrak{z}tp) \supseteq \Gamma(t)$ for all $t \in \delta$.

4.19. Example

Consider $\delta = \{\eta, \rho, \tau, \partial\}$ and define "." on δ such that

$\cdot$	η	ρ	τ	∂
η	η	η	ρ	∂
ρ	η	ρ	ρ	∂
τ	η	ρ	τ	∂
∂	∂	∂	∂	∂

Then $(\delta, .)$ is semigroup. We define relation "Ω"on δ as follows:

$\Omega = \{(\eta,\eta), (\eta,\rho), (\rho,\rho)(\tau,\tau)(\partial,\partial)\}$.

Then $(\delta, ., \Omega)$ is "POsG". Now we define "SPOsG" over δ such that $\Gamma(\eta) = \{\eta, \rho, \partial\}$, $\Gamma(\rho) = \{\eta, \rho, \partial\}$, $\Gamma(\tau) = \{\partial\}$ and $\Gamma(\partial) = \delta$.

Now we prove the condition of "restricted double interior soft left ideal", for this we prove for $\mathfrak{g}, \vartheta \in \delta$ such that $\mathfrak{g}\Omega\vartheta \Rightarrow \Gamma(\mathfrak{g}) \subseteq \Gamma(\vartheta)$ where $\Gamma(\mathfrak{g})$ is L-ideal of $\Gamma(\vartheta)$ and $\Gamma(\mathfrak{g}p\vartheta) \supseteq \Gamma(p)$.

Further

$\Gamma(\eta\eta\eta) = \Gamma(\eta) = \{\eta, \rho, \partial\} \supseteq \{\eta, \rho, \partial\} = \Gamma(\eta)$
$\Rightarrow \Gamma(\eta\eta\eta) \supseteq \Gamma(\eta)$.

$\Gamma(\eta\rho\eta) = \Gamma(\eta) = \{\eta, \rho, \partial\} \supseteq \{\eta, \rho, \partial\} = \Gamma(\rho)$
$\Rightarrow \Gamma(\eta\rho\eta) \supseteq \Gamma(\rho)$.

$\Gamma(\eta\tau\eta) = \Gamma(\eta) = \{\eta, \rho, \partial\} \supseteq \{\partial\} = \Gamma(\tau)$
$\Rightarrow \Gamma(\eta\tau\eta) \supseteq \Gamma(\tau)$.

$\Gamma(\eta\partial\eta) = \Gamma(\partial) = \delta \supseteq \delta = \Gamma(\partial)$
$\Rightarrow \Gamma(\eta\partial\eta) \supseteq \Gamma(\partial)$.

$\Gamma(\eta\eta\rho) = \Gamma(\eta) = \{\eta, \rho, \partial\} \supseteq \{\eta, \rho, \partial\} = \Gamma(\eta)$
$\Rightarrow \Gamma(\eta\eta\rho) \supseteq \Gamma(\eta)$.

$\Gamma(\eta\rho\rho) = \Gamma(\eta) = \{\eta, \rho, \partial\} \supseteq \{\eta, \rho, \partial\} = \Gamma(\rho)$
$\Rightarrow \Gamma(\eta\rho\rho) \supseteq \Gamma(\rho)$.

$\Gamma(\eta\tau\rho) = \Gamma(\eta) = \{\eta, \rho, \partial\} \supseteq \{\eta, \rho, \partial\} = \Gamma(\tau)$
$\Rightarrow \Gamma(\eta\tau\rho) \supseteq \Gamma(\tau)$.

$\Gamma(\eta\partial\rho) = \Gamma(\partial) = \delta \supseteq \delta = \Gamma(\partial)$
$\Rightarrow \Gamma(\eta\partial\rho) \supseteq \Gamma(\partial)$.

$\Gamma(\rho\eta\rho) = \Gamma(\eta) = \{\eta, \rho, \partial\} \supseteq \{\eta, \rho, \partial\} = \Gamma(\eta)$
$\Rightarrow \Gamma(\rho\eta\rho) \supseteq \Gamma(\eta)$.

$\Gamma(\rho\rho\rho) = \Gamma(\rho) = \{\eta, \rho, \partial\} \supseteq \{\eta, \rho, \partial\} = \Gamma(\rho)$
$\Rightarrow \Gamma(\rho\rho\rho) \supseteq \Gamma(\rho)$.

$\Gamma(\rho\tau\rho) = \Gamma(\rho) = \{\eta, \rho, \partial\} \supseteq \{\partial\} = \Gamma(\tau)$
$\Rightarrow \Gamma(\rho\tau\rho) \supseteq \Gamma(\tau)$.

$\Gamma(\rho\partial\rho) = \Gamma(\partial) = \delta \supseteq \delta = \Gamma(\partial)$
$\Rightarrow \Gamma(\rho\partial\rho) \supseteq \Gamma(\rho)$.

$\Gamma(\tau\eta\tau) = \Gamma(\rho) = \{\eta, \rho, \partial\} \supseteq \{\eta, \rho, \partial\} = \Gamma(\eta)$
$\Rightarrow \Gamma(\tau\eta\tau) \supseteq \Gamma(\eta)$.

$\Gamma(\tau\rho\tau) = \Gamma(\rho) = \{\eta, \rho, \partial\} \supseteq \{\partial\} = \Gamma(\tau)$
$\Rightarrow \Gamma(\tau\rho\tau) \supseteq \Gamma(\tau)$.

$\Gamma(\tau\tau\tau) = \Gamma(\tau) = \{\partial\} \supseteq \{\partial\} = \Gamma(\tau)$
$\Rightarrow \Gamma(\tau\tau\tau) \supseteq \Gamma(\tau)$.

$\Gamma(\tau\partial\tau) = \Gamma(\partial) = \delta \supseteq \delta = \Gamma(\tau)$
$\Rightarrow \Gamma(\tau\partial\tau) \supseteq \Gamma(\tau)$.

$\Gamma(\partial\eta\partial) = \Gamma(\partial) = \delta \supseteq \{\eta, \rho, \partial\} = \Gamma(\eta)$
$\Rightarrow \Gamma(\partial\eta\partial) \supseteq \Gamma(\eta)$.

$\Gamma(\partial\rho\partial) = \Gamma(\partial) = \delta \supseteq \{\eta, \rho, \partial\} = \Gamma(\rho)$
$\Rightarrow \Gamma(\partial\rho\partial) \supseteq \Gamma(\rho)$.

$\Gamma(\partial\tau\partial) = \Gamma(\partial) = \delta \supseteq \{\eta, \rho, \partial\} = \Gamma(\tau)$
$\Rightarrow \Gamma(\partial\tau\partial) \supseteq \Gamma(\tau)$.

$\Gamma(\partial\partial\partial) = \Gamma(\partial) = \delta \supseteq \delta = \Gamma(\partial)$
$\Rightarrow \Gamma(\partial\partial\partial) \supseteq \Gamma(\partial)$.

$\Rightarrow (\Gamma, \delta)$ is "RDTS-Interior L-ideal".

4.20. Definition

A "SPOsG" (Γ,δ) is indicated to be a "RDTS-Interior ideal" over $\mathcal{S}$ if it follows both assertions of "RDTS-Interior L-ideal" and "RDTS-Interior R-ideal".

4.21. Theorem

Each "DTS-Interior L-ideal" (resp. "DTS-Interior R-ideal") is "RDTS-Interior L-ideal" (resp. "RDTS-Interior R-ideal").

Proof:

Let $(\delta,.,\Omega)$, $(\mathcal{S},.,\partial)$ be two "POsG" and (Γ,δ) be a "DTS-Interior L-ideal" (resp. "DTS-Interior R-ideal") over $\mathcal{S}$. To prove (Γ,δ) is "RDTS-Interior L-ideal" (resp. "RDTS-Interior R-ideal") over $\mathcal{S}$. As (Γ,δ) is a "DTS-Interior L-ideal" (resp. "DTS-Interior R-ideal") over δ so $\forall\, t,b,p\in\delta$ such that $\Gamma(b)\subseteq\Gamma(tbp)$ and if $t\Omega p$ implies that $\Gamma(t)\subseteq\Gamma(p)$ Where $\Gamma(t)$ is "L-ideal" (R-ideal) of $\Gamma(p)$. Now we can say that $\forall\,(t,p)\in\Omega$, $\Gamma(t)\subseteq\Gamma(p)$ (Where Γ(t) is "L-ideal" (R-ideal) of Γ(p)). Since $\mathfrak{z}\Omega p$ then $\Gamma(b)\subseteq\Gamma(tbp)$ implies that (Γ,δ) is "RDTS-Interior L-ideal" (resp. "RDTS-Interior R-ideal") over $\mathcal{S}$.

4.22. Definition

Let $(\delta,.,\Omega)$ and $(\mathcal{S},.,\partial)$ be two "POsG". Then a "SPOsG" (Γ,δ) is indicated to be a "St-DTS-Interior R-ideal" (resp. "St-DTS-Interior L-ideal") over $\mathcal{S}$ if

(1) $\Gamma(\mathfrak{z}tp)\supseteq\Gamma(t)\ \forall\,\mathfrak{z},t,p\in\delta$, Where $\Gamma(t)$ is "L-ideal" (R-ideal) of $\Gamma(\mathfrak{z}tp)$.

(2) $\mathfrak{z}\Omega p$ then $\Gamma(\mathfrak{z})\subseteq\Gamma(p)$ Where $\Gamma(\mathfrak{z})$ is "L-ideal" (R-ideal) of $\Gamma(p)$.

4.23. Definition

A "SPOsG" (Γ,δ) is indicated to be a "St-DTS-int-ideal" over $\mathcal{S}$ if it follows both assertions of "St-DTS-Interior L-ideal" and "St-DTS-Interior R-ideal".

4.24. Definition

Let $(\delta,.,\Omega)$ and $(\mathcal{S},.,\partial)$ be two "POsG". Then a "SPOsG" (Γ,δ) is indicated to be a "RSt-DTS-Interior L-ideal" ("RSt-DTS-Interior R-ideal") over $\mathcal{S}$ if for $\mathfrak{z},p\in\delta$

(1) $\mathfrak{z}\Omega p$ then $\Gamma(\mathfrak{z})\subseteq\Gamma(p)$, Where $\Gamma(\mathfrak{z})$ is "L-ideal" (R-ideal) of $\Gamma(p)$.

(2) Now $\mathfrak{z}\Omega p$ then $\Gamma(\mathfrak{z}tp)\supseteq\Gamma(t)$ for all $t\in\delta$, Where $\Gamma(t)$ is "L-ideal" (R-ideal) of $\Gamma(\mathfrak{z}tp)$.

Note that, example 4.15 also represent a "RSt-DTS-Interior L-ideal".

4.25. Definition

A "SPOsG" (Γ,δ) is indicated to be a "RSt-DTS-int-ideal" over $\mathcal{S}$ if it follows both assertions of "RSt-DTS-Interior L-ideal" and "RSt-DTS-Interior L-ideal".

4.26. Theorem

Each "St-DTS-Interior L-ideal" ("St-DTS-Interior R-ideal") is "RSt-DTS-Interior L-ideal" ("RSt-DTS-Interior R-ideal").

Proof:

Let $(\delta,.,\Omega)$, $(\mathcal{S},.,\partial)$ be two "POsG" and (Γ,δ) be a "St-DTS-Interior L-ideal" ("St-DTS-Interior R-ideal") over $\mathcal{S}$. To prove (Γ,δ) is "RSt-DTS-Interior L-ideal" ("RSt-DTS-Interior R-ideal") over $\mathcal{S}$. As (Γ,δ) is a "St-DTS-Interior L-ideal" ("St-DTS-Interior R-ideal") over δ so $\forall\, t,b,p\in\delta$ such that $\Gamma(b)\subseteq\Gamma(tbp)$(where Γ(b) is "L-ideal" (R-ideal) of Γ(tbp)) and if $t\Omega p$ implies that $\Gamma(t)\subseteq\Gamma(p)$ Where $\Gamma(t)$ is "L-ideal" (R-ideal) of $\Gamma(p)$. Now we can say that $\forall\ (t,p)\in\Omega$, $\Gamma(t)\subseteq\Gamma(p)$ Where $\Gamma(t)$ is "L-ideal" (R-ideal) of $\Gamma(p)$, since $\mathfrak{z}\Omega p$ then $\Gamma(b)\subseteq\Gamma(tbp)$ $(\forall\, b\in\delta)$ where $\Gamma(b)$ is "L-ideal" (R-ideal) of $\Gamma(tbp)$ implies that (Γ,δ) is "RSt-DTS-Interior L-ideal" ("RSt-DTS-Interior R-ideal") over $\mathcal{S}$.

4.27. Proposition

Each "St-DTS-Interior L-ideal" ("St-DTS-Interior R-ideal") is "DTS-Interior L-ideal" ("DTS-Interior R-ideal").

Proof:

Let$(\delta,.,\Omega)$, $(\mathcal{S},.,\partial)$ be two "POsG" and (Γ,δ) be a "St-DTS-Interior L-ideal" ("St-DTS-Interior R-ideal") over$\mathcal{S}$. To prove (Γ,δ) is "DTS-Interior L-ideal" ("DTS-Interior R-ideal") over $\mathcal{S}$. As (Γ,δ) is a "St-DTS-Interior L-ideal" ("St-DTS-Interior R-ideal") over δ so $\forall\, t,b,p\in\delta$ such that $\Gamma(b)\subseteq\Gamma(tbp)$ where $\Gamma(b)$ is "L-ideal" (R-ideal) of $\Gamma(tbp)$ and if $t\Omega p$ implies that $\Gamma(t)\subseteq\Gamma(p)$ Where $\Gamma(t)$ is "L-ideal" (R-ideal) of $\Gamma(p)$. Then we can say $\forall\, t,b,p\in\delta$ such that

$\Gamma(b) \subseteq \Gamma(tbp)$ and if $t\Omega p$ implies that $\Gamma(t) \subseteq \Gamma(p)$ Where $\Gamma(t)$ is "L-ideal" (R-ideal) of $\Gamma(p)$ implies that (Γ, δ) is "DTS-Interior L-ideal" ("DTS-Interior R-ideal") over S.

4.28. Corollary

Each "St-DTS-Interior L-ideal" ("St-DTS-Interior R-ideal") is "RDTS-Interior L-ideal" ("RDTS-Interior R-ideal").

4.29. Corollary

Each "RSt-DTS-Interior L-ideal" ("RSt-DTS-Interior R-ideal") is "RDTS-Interior L-ideal" ("RDTS-Interior R-ideal").

Conclusion

We generalized the concept of soft ideals of po-semi group in terms of twisted soft ideal, restricted twisted soft ideals, twisted soft bi-ideals, restricted soft bi-ideal, twisted soft interior ideal, strong double twisted soft interior ideal. Further this concept can be extended to fuzzy ideal by using concept of fuzzy set theory.

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