

## Population Density Pattern Using Spatial Interaction Models and its Simulation

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### ABSTRACT

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*This Communication attempts to examine the Urban Population Density of Karachi using Spatial Interaction models. It determines urban densities and analyzes density models and relates them with those from empirical analyses of real urban data. In view of these models new spatial structures are proposed for the existing urban areas. Considering various hypothetical ring shaped areas surrounding our pre-selected centre the spatial structure of the city is discussed. The city is divided into 10, 15 and 20 such rings (with radii increasing by 1). Then the Flatten Gradient Density Model **FGDM** developed in Abbas [Shaheen Abbas] is applied on these concentric rings to determine population density to compute various gradient parameters. Using these computations best suited value of the gradient parameter  $\beta$  is determined. The best suited value of  $\beta$  appears to be 1 for which the density turns out to be 1. Linear Model, Log Linear Model and Log-Transformation Exponential Models **LETM** are used to verify the results. The best correlation is obtained by **LTEM**.*

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**Keyword:** Urban Population Density of Karachi (UPDK), Gravity Base (GB), Flattened Gradient Density Model (FGDM), Spatial Interaction Models (SIM).

## 1. Introduction

The Direct manifestation of Karachi's changing population structure increases in gross population density. This pattern of urban development suggests that there are still substantial opportunities for urban development. The erstwhile Karachi Development Authority (KDA) and other agencies have already developed the area from 10 to 30+ kilometers calculated from the centre of the city. Directorate of Urban Planning of Karachi Metropolitan Corporation in a study entitled SPOT (1985-2004) based on satellite imagery [26] for the core area (within five kilometers from the city center) found that the population density has increased by 366 percent (from 75 to 105 persons per acre). It indicated that there has been a gradual increase in filling and densification of land areas in the center. In the inner urban city between 5 to 10 kilometers the density doubled in the duration (1972-1997). And in the range of 5 to 20 kilometers out from the center of the city from 1972 to 2012 the density nearly tripled. The corresponding increase was observed from 23 to 42 and finally to 69 persons per acre. The suburban zones (lying between 10 and 20 kilometers from the center) the population density also nearly tripled and in the rural fringes (beyond 20 kilometers from the center) the population density increased even more. Meanwhile, many rural areas were converted into sub-urban areas and finally were included into urban areas. It is to mention that roads, amenity plots and common open lands for parks or spaces for public use are not included in the calculations [26], (MP/RR/13 Master plan 1961-2001, Report Karachi Development Plan 1974-85, 1985).

According to master plan of the old KDA estimates, 22,264 acres of land was needed for urban expansion in 6-years. 45 % (10,200 acres) of this vacant developed land was located between 10 and 30 kilometers from the center and the region was expected to grow by about 500,000 persons per year [26]. If we apply the ratio of population growth to urban land on the version of 74 persons per acre, the annual increase in urban land conversion will be 6,780 acres per year. However, the population explosion has ruled out all the estimates. Over 40 percent of the land conversion took place in

the band between 10 and 20 kilometers from the city center. The next two rings, from 20 to 30 kilometers in distance, account for 16 percent of the urban land conversion. Finally, the last area, on the fringe, accounted for 29 percent of the increase in urban land. The principal reason for such a high proportion is the development of the Pakistan Steel Mills in the fringes of eastern Karachi. This facility accounts for 10,447 acres out of the 16,309 urbanized acres in the peripheral area. However, despite this pattern of decentralization, according to a study of [25], [26] in 1997, centralization still persists. It shows that centralized planned areas are 22 percent of all planned residential areas and falls within 15 to 30 kilometers radius of the CBD. The study suggested that for each additional 74 persons, an additional acre is needed for urban use. In the same duration an increase in the land use for industrial purposes took place (it increased from 4,858 to 25,976 kilometers). Subsequently it indicated an enormous growth in the economy of Karachi as well (spurring employment and the migration of job seekers and their families) (UNDP Project PAK/2000/029, Shaheen Abbas, M. Rashid Kamal 2010, MPK/RR/24 1992).

## **2. Material and Methods**

The population and area data of Karachi obtained from 1998 census is arranged using Microsoft EXCEL. The city was then distributed in circular rings of radius 10, 15 and 20 kilometers. ARCVIEW (3.2) was used to calculate distances and areas and to compute population density.

Simulation using dynamical CLUE and gravity based GB models was performed to establish population and distance relationship and to represent the spatial pattern. These representations were applied to visualize the patterns of the city of Karachi. Data so generated was used to establish FGDM. By varying the travelling fractional coefficient  $\beta$ , various urban density patterns for the city in FGDM developed by [25], [26] were found. The parameter  $\beta$  varied through 2.0, 1.5, 1.0, 0.5 and 0.2. The empirical characterization of location suitability was confirmed via LRM, LTEM and LLM.

## 2.1 Land Conversion Models

This section discusses some land use models. In case of FGDM [11], [26], the density at a distance  $r$  from a pivot is given as  $D(r) = D_0 e^{-\beta r}$ , for different  $\beta$ . Mono and multi pivotal techniques for FGDM were developed in [25], [26] where the technique was employed to study population density patterns of Katchi Abadies of Karachi. Here the population density pattern of the whole city will be studied using multi-pivotal technique.

## 2.2 Spatial Interaction Models

Spatial Interaction Models (SIM) were developed by Wang and Goldmann [11]. These models are capable of studying urban flow patterns. CLUE model and various gravity based models such as Garin-Lowry model (Potential model) are in general termed as spatial interaction models. These models study land use variations. They empirically compute the relation among land use and its variations [21]. CLUE model was applied to investigate upcoming variations in the spatial pattern of the land [29]. Henderson and, Mookherjee [9] [20], used the model for the study of urban fringes in China, whereas, Abbas [26], used these models for the fringes in Karachi. In fact, it is a Dynamical modeling linking Spatial representation of the city under consideration and Empirical characterization of location suitability.

Gravity models are analogous to the Newton's gravity model. These were further improved by applying the entropy-maximizing principles [3, 26, 2, 15, 12, 28, 22]. The gravity-based models are used to overcome limitations of the usual urban models (e.g. radial exchange in the direction of the pivot), hence are most suitable for the purpose used in [30]. This study uses Garin-Lowry model (also known as potential model) which is described as

$$v_j = b \sum_i \frac{P_i}{d_{ij}^\beta} \quad (1)$$

where,  $v_j$  is the potential point,  $b$  is constant multiplier,  $P_i$  population size of location  $i$ ,  $d_{ij}$  distance between the population at

$p_i$  and population at point  $p_j$  and  $\beta$  is distance friction parameter. As mentioned, Garin-Lowry model is a special type of GB model known as potential model. Potential model measure the force applied by a set of population situated at a certain point in space [30, 26, 22, 1, 8,]. The potential at point  $j$  is represented by

$$p_j = c \left( b \sum_{i=1}^n \frac{P_i}{d_{ij}^\beta} \right) \quad (2)$$

where  $p_j$  is population in cell  $j$ ,  $b$  is constant multiplier,  $c$  is unknown constant  $p_i$  is population size,  $d_{ij}$  is separation among the population  $p_i$  and point  $j$  and  $\beta$  is distance friction factor,

$$k = \frac{1}{bc}$$

Eq. (1) is rewritten as

$$kp_j = \sum_{i=1}^n \frac{P_i}{d_{ij}^\beta} \quad (3)$$

Eq. (3) in matrix form is representad as

$$kP = AP \quad (4)$$

Where,  $P$  is column vector of  $n$  elements ( $p_1, p_2, \dots, p_n$ ),  $A = n \times n$  matrix involving  $d_{ij}^\beta$ ,  $k$  is unknown scalar and  $\beta$  is distance friction parameter.

Clearly equation (3) and equation (4) represent systems of  $n$  number of nonlinear equations with  $(n + 1)$  unknowns ( $n$ ,  $p_i$ 's and 1 constant  $k$ ). Spatial change of population density are studied here with cell size computed with a  $p$  that normalizes the population (e.g.  $p_1=1$ ). This selection will resolve the system equations [30].

## 2.4 The Urban Spatial Structure

Curry [15], Barras and Richard [1], Barnsley [28] developed a potential model to study the virtual spatial structure of the fringes of a city using mono-pivots. Here it is used to study the structure of the whole city (in fact a megacity) using multi-pivots [Abbas 2010]. For this purpose we will divide the city into  $n \times m$  equal-

area cells with the help of  $n$  rings (of radii 1, 2, ...  $n + 1$  units) and  $m$  sectors, as Fig: 1.

Given that the area is equally divided into  $n$  regions, each cell has an area divided by  $m$ . The distance from the city center to the center point of the  $i$ th ring is

$$T_i = \frac{r_i + r_{i-1}}{2} = \frac{\sqrt{i}}{2} + \frac{\sqrt{i-1}}{2} \quad (5)$$

Number of rings can be increased according to the increase in the size of the city. The population at a given distance from the city center is assumed to be the same regardless of the direction. As all cells within a given ring have the same population, it is needed only to differentiate population size across the rings. A cell corresponding to a ring  $i$  ( $i = 1, 2, \dots, n$ ) and a sector  $s$  ( $s = 1, 2, \dots, m$ ) as in Fig:1 will be represented as  $C_{ij}$ . The distance between a cell  $C_{ij}$  and any other cell  $C_{is}$  is denoted by  $d_i(j, s)$  and is calculated according to the shortest path rule. By using the Fig: 2.  $d_n(2,3)$  is the distance between the  $n$ th reference cell and the cell in the second ring of the third sector, as indicated by the dashed line path. In any ring, there are  $m$  identical cells, for  $s = 1, 2, \dots, m$ . Using equation (3) it is used a multi-pivotal model instead of the mono-pivotal model. Such mono-pivotal models were earlier used by [3], [30] and [16](Clarke 1972, whereas, [26] used mono and multi pivotal models both in a different perspective. The model equations are as follows.

$$\sum_{i=1}^n \left( \sum_{s=1}^m \frac{1}{d_j^\beta(j, s)} \right) p_i \quad (6)$$

## 2.5 Proposed GB Model for Karachi

Referring to Figure 2, Karachi is divided in to 2 rings ( $n = 2$ ) and 4 sectors so that  $m = 4$  giving 8 equal-sized cells. The cells  $C_{11}, C_{12}, C_{13}, C_{14}$ , each having a population  $p_1$  and the cells  $C_{21}, C_{22}, C_{23}, C_{24}$ , each having a population  $p_2$ . The  $C_{11}$  and  $C_{21}$  are chosen as reference cells. The distances are calculated by using the algorithm given by [30]; however, they used monopivotal technique,

whereas, [26], used both the mono and multi pivotal techniques. The details are given in the following.

- i. When  $j > i$ , the distance  $d_{j(i,s)}$  between a reference cell  $C_{p1}$  ( $p = 1, 2$ ) and any other cell  $C_{is}$  consists of the line EF and the arc FG (Fig: 3) and is given as

$$d_{j(i,s)} = (T_j - T_i) + 2\Pi T_i (s-1)/m \quad (7)$$

where  $T_j$  and  $T_i$  are defined by equation (5). The distance from the city center to the center point of the  $i$ th ring, an area divided by  $m$  and  $T_j = 1 \dots 20$  maximum size of ring range and  $T_i = 1$  minimum size of range are shown number of rings can be increased in the size of the city. A cell corresponding to a ring  $i$  ( $i = 1, 2, \dots, n$ ) and a sector  $s$  ( $s = 1, 2, \dots, m$ ). This distance formula also applies when  $j = i$ , except when  $s = 1$ .

- ii. When  $j < i$ ,  $d_{j(i,s)}$  takes the inner (shorter) arc EF and line FG (Fig:1.4). Then  $d_{j(i,s)}$  is equivalent to  $d_i(j,s)$  and formula (1.7) can be rewritten as

$$d_{j(i,s)} = (T_i - T_j) + 2\Pi T_j (s-1)/m \quad (8)$$

- iii. Finally, we consider the special case of measuring the distance  $d_{j(i,s)}$  when  $i = j$  and  $s = 1$ , or the distance within a reference cell itself (as Fig: 5).

$$d_{j(i,s)} = 0.5 \times 0.5 \times (r_i - r_{i-1} + 2\Pi T_i / m) \quad \dots (9)$$

For calculating the distance between a reference cell  $C_{p1}$  ( $p = 1, 2$ ) and any other cell (Fig: 2) we will follow the route through the shortest arc. The radii of the circles are as described earlier.

In distance travel friction  $\beta = 0.2, 0.5, 1.5, 2.0$ . In particular  $\beta = 2.0$ , Using the above mentioned radii and taking  $\beta = 2.0$ , equation (6) gives

$$k p_1 = 9.3110 p_1 + 3.1410 p_2 \quad \dots (10)$$

$$k p_2 = -3.1410 p_1 + 3.7254 p_2 \quad \dots (11)$$

Normalizing  $p_2 = 1$ ,

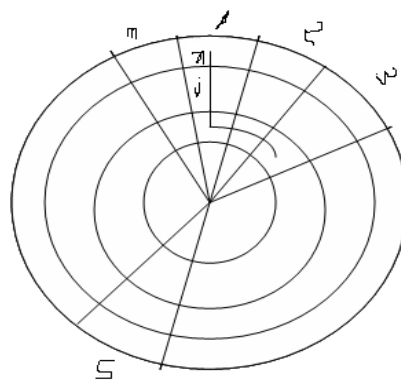
$$k p_1 = 9.3110 p_1 + 3.1410 \quad \dots (12)$$

$$k = -3.1410 p_1 + 3.7254 \quad \dots (13)$$

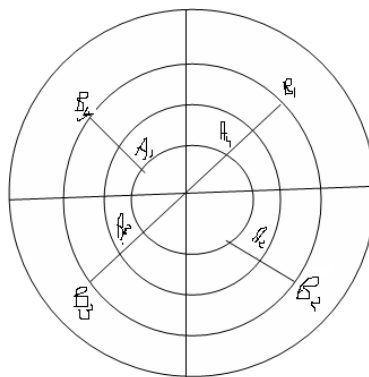
We thus obtain a system of two equations in two variables  $p_1$  and  $k$  which can be further reduced to the quadratic equation in  $k$  only.

$$k^2 = 12.987k - 45.0075 \quad \dots (14)$$

Solving for  $k$  yields  $k = 10.7558$  and  $2.2313$ . As population size cannot be negative, only the value  $k = 10.7558$  yields an acceptable solution  $p_1 = 2.1692$ . GB model are indicated to increasing number of ring city size is decreased.

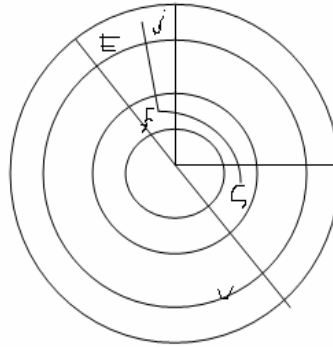


**Figure: 1** Urban Spatial Structures

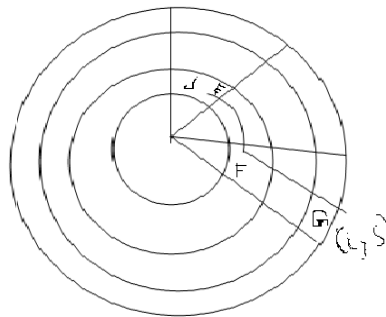


**Figure: 2** Simplified Urban Spatial Structure

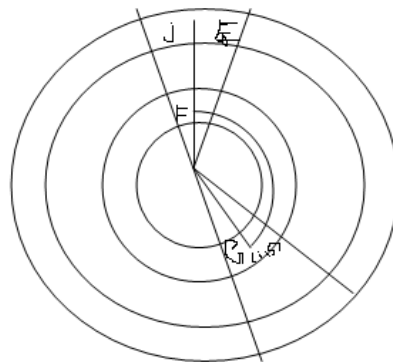




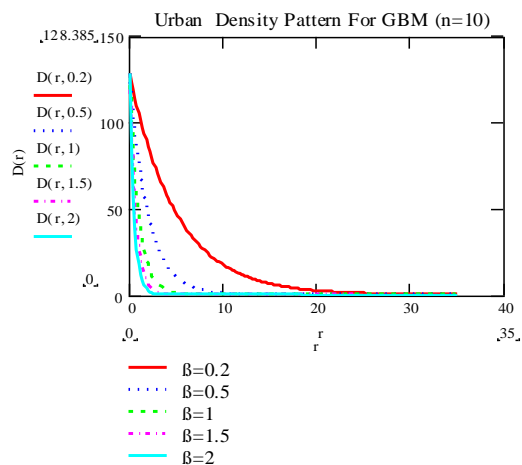
**Figure:3** Distance  $d_j(i,s)$ , when  $j > i$



**Figure: 4** Distance  $d_j(i,s)$ , when  $j < i$



**Figure: 5** Distance  $d_j(i,s)$  When  $s > m/\Pi+1$



**Figure: 6** Urban Density Pattern for GBM (n=10)

### 3. Results and Discussion

Using population calculated by GB Model, the density patterns are studied by applying Pivotal technique of FGDM. The results are presented in Tables: 1 to 5. Increasing the radius and accordingly varying the number of rings (from 10, 15, 20) and decreasing  $\beta$ , the density gradient declined consistently (Fig: 6). This means that the density flattens. Observing Table: 1.1 for the case  $n = 10$ , for all the values of  $\beta$  (2.0, 1.5, 1.0, 0.5 and 0.2) the population density goes on decreasing as we move from ring 1 to ring 10. In particular the population density becomes 1 for all values of  $\beta$  in ring 10. In the same way we repeat the method moving from ring 1 to ring 15 and from ring 1 to ring 20 varying  $\beta$  from 2.0 to 0.2 we observe that the population density flattens as we increase the number of rings. For  $\beta = 1$  (Table 3 (b)) the population density comes out to be 1 irrespective of the number of rings which indicates a good distribution of population. The above result shows a flattening of distance-density curves. The inherent linearity in the FGDM can be gauged from the Linear Regression Model (LRM) ( $y = \alpha + \beta x$ ), Linear Transformation Exponential Model (LTEM) ( $\ln y = \alpha + \beta x$ ) and Log Linear Model (LLM) ( $\ln y = \alpha + \beta \ln x$ )

all indicate a good distance-density correlation. These results are depicted in Table: 6 to 8. For the various city sizes i. e. for  $n = 10, 15$  and  $20$  and for  $\beta$  values of  $2.0, 1.5, 1.0, 0.5, 0.2$  we observe that for the LRM the largest  $R^2$  value exceeds  $0.99$  for  $\beta = 0.5$  and  $2.0$  and in case of LTEM the largest  $R^2$  value exceeds  $0.97$  for  $\beta = 1.0, 0.5, 0.2$ , while for the LLM the largest  $R^2$  value exceeds  $0.99$  for  $\beta = 0.5$  and  $2.0$ . In both the cases as the gradient  $\beta$  goes on decreasing, the density-distance curves go on flattening.

**Table 1:**Urban Densities of various  $\beta$  values  $n= (10)$

Ring (j)	Distance r	D(r) $\beta=2.0$	D(r) $\beta=1.5$	D(r) $\beta=1.0$	D(r) $\beta=0.5$	D(r) $\beta=0.2$
1	0.5	37.72	7.12	2.12	1.39	1.13
2	1.207	4.18	2.16	1.58	1.25	1.09
3	1.573	2.86	1.85	1.46	1.2	1.07
4	1.866	2.28	1.68	1.38	1.16	1.06
5	2.118	1.94	1.56	1.31	1.13	1.05
6	2.343	1.71	1.46	1.25	1.11	1.04
7	2.548	1.54	1.37	1.2	1.08	1.03
8	2.737	1.39	1.28	1.14	1.05	1.02
9	2.914	1.23	1.17	1.08	1.03	1.01
10	3.081	1	1	1	1	1

**Table 2:** Urban Density of various City size for  $\beta=2.0$

Ring	Distance	n=10	n=15	n=20
(j)	r	D(r)	D(r)	D(r)
1	0.5	35.72	35.72607	35.73213
2	1.207	4.18	4.200937	4.221873
3	1.573	2.86	2.878668	2.906472
4	1.866	2.28	2.295474	2.317374
5	2.118	1.94	1.95323	1.97281
6	2.343	1.71	1.722485	1.73899
7	2.548	1.54	1.550953	1.566076
8	2.737	1.39	1.400362	1.414277
9	2.914	1.23	1.239766	1.253873
10	3.081	1	1.010101	1.011663
11	3.24		1.408773	1.422374
12	3.39		1.016605	1.027097
13	3.535		0.746254	0.757104
14	3.674		0.557377	0.567115
15	3.807		0.419616	0.428262
16	3.936			0.328185
17	4.062			0.254495
18	4.183			0.198947
19	4.301			0.155469
20	4.416			0.123441

**Table 3 (a)** Urban Density of various City size for  $\beta=1.5$

Ring	Distance	n=10	n=15	n=20
(j)	r	D(r)	D(r)	D(r)
1	0.5	5.12	7.43443	7.713658
2	1.207	2.16	33.72621	34.54382
3	1.573	1.85	79.10333	81.1447
4	1.866	1.68	157.1251	162.4333
5	2.118	1.56	283.0594	295.0197
6	2.343	1.46	482.7908	503.7817
7	2.548	1.37	782.6375	823.5215
8	2.737	1.28	1217.877	1283.455
9	2.914	1.17	1837.409	1968.653
10	3.081	1	2700.928	2922.316
11	3.24		3920.417	4249.864
12	3.39		5512.436	6087.647
13	3.535		7645.404	8540.812
14	3.674		10432.26	11894.73

15	3.807		12035.34	16178.74
16	3.936			21772.87
17	4.062			29062.9
18	4.183			38284.87
19	4.301			29551.55
20	4.416			62317.65

**Table 3: (b)** Urban Density of various City size for  $\beta=1.0$ 

Ring	Distance	n=10	n=15	n=20
(j)	r	D(r)	D(r)	D(r)
1	0.5	2.12	2.13	2.21
2	1.207	1.58	1.65	1.69
3	1.573	1.46	1.55	1.59
4	1.866	1.38	1.48	1.53
5	2.118	1.31	1.42	1.48
6	2.343	1.25	1.38	1.44
7	2.548	1.2	1.34	1.41
8	2.737	1.14	1.3	1.37
9	2.914	1.08	1.26	1.35
10	3.081	1	1.22	1.32
11	3.24		1.19	1.29
12	3.39		1.15	1.27
13	3.535		1.11	1.24
14	3.674		1.07	1.22
15	3.807		1	1.19
16	3.936			1.16
17	4.062			1.13
18	4.183			1.1
19	4.301			1.06
20	4.416			1

**Table 4:** Urban Density of various City size for  $\beta=0.5$ 

Ring	Distance	n=10	n=15	n=20
(j)	r	D(r)	D(r)	D(r)
1	0.5	4.48804	4.50921	4.67857
2	1.207	9.65335	10.08728	10.33182
3	1.573	15.45506	16.40777	16.8321
4	1.866	22.67093	24.31375	25.13516
5	2.118	31.40689	34.0441	35.48259
6	2.343	41.99891	46.36679	48.38274

7	2.548	54.83461	61.23198	64.43067
8	2.737	69.1673	78.875	83.12211
9	2.914	101.6464	99.69462	106.8157
10	3.081		124.0086	134.1732
11	3.24		153.5388	166.4412
12	3.39		185.8169	205.2065
13	3.535		222.9305	249.0395
14	3.674		264.7163	301.8261
15	3.807		218.2	359.4026
16	3.936			425.1406
17	4.062			500.3049
18	4.183			583.947
19	4.301			671.6712
20	4.416			752.9509

**Table 5:** Urban Density of various City size for  $\beta=0.2$

Ring	Distance	n=10	n=15	n=20
(j)	r	D(r)	D(r)	D(r)
1	0.5	3.862892	3.881113	4.026883
2	1.207	6.724952	7.022893	7.193145
3	1.573	9.641112	10.23543	10.49957
4	1.866	12.95243	13.98101	14.3603
5	2.118	16.63697	18.03396	18.79596
6	2.343	20.79564	22.95839	23.95658
7	2.548	25.53172	28.51042	29.99977
8	2.737	30.24998	34.70085	36.56936
9	2.914	35.65042	41.59216	44.56303
10	3.081	40.33421	49.20774	53.24116
11	3.24		58.08766	62.96898
12	3.39		67.20596	74.21876
13	3.535		77.19696	86.23804
14	3.674		87.92276	100.2484
15	3.807			114.7038
16	3.936			130.5322
17	4.062			174.912
18	4.183			166.4858
19	4.301			184.836
20	4.416			200.1766

**Table 6:** Regression Analysis for LRM

City size	$\beta$ value	a	b	$R^2$
n=10	2	25.75	-9.75	0.552
	1.5	4.5	-1.26	0.742
	1	2.15	-0.38	0.942
	0.5	1.43	-0.14	0.99
	0.2	1.15	-0.05	0.997
n=15	2	20.9	-6.42	0.435
	1.5	3.56	-0.72	0.683
	1	2.07	-0.28	0.941
	0.5	1.43	-0.11	0.99
	0.2	1.15	-0.04	0.997
n=20	2	13.63	-3.51	0.347
	1.5	2.89	-0.43	0.621
	1	2.01	-0.22	0.943
	0.5	1.43	-0.1	0.991
	0.2	1.15	-0.03	0.996

**Table 7:** Regression Analysis for LTEM

City size	$\beta$ value	a	b	$R^2$
n=10	2	3.32	-1.16	0.845
	1.5	1.61	-0.53	0.903
	1	0.83	-0.26	0.978
	0.5	0.38	-0.12	0.995
	0.2	0.15	-0.04	0.998
n=15	2	2.83	-0.78	0.763
	1.5	1.32	-0.33	0.852
	1	0.78	-0.2	0.974
	0.5	0.37	-0.1	0.995
	0.2	0.15	-0.04	0.998
n=20	2	2.13	-0.49	0.592
	1.5	1.07	-0.21	0.772
	1	0.75	-0.16	0.971
	0.5	0.37	-0.08	0.994
	0.2	0.15	-0.03	0.997

**Table 8:** Regression Analysis for LLM

City size	$\beta$ value	a	b	R <sup>2</sup>
n=10	2	2.07	-1.85	0.977
	1.5	1.03	-0.82	0.982
	1	0.52	-0.38	0.962
	0.5	0.24	-0.17	0.943
	0.2	0.09	-0.07	0.939
n=15	2	2.14	-0.57	0.948
	1.5	1	-0.63	0.957
	1	0.57	-0.35	0.973
	0.5	0.27	-0.17	0.926
	0.2	0.1	-0.07	0.924
n=20	2	1.87	-1.22	0.851
	1.5	1.31	-0.48	0.909
	1	0.6	-0.32	0.928

### 3. Conclusion

In this study spatial interaction models have been used to explore the near future changes in the Spatial pattern of the land use in urban Karachi which is important from the point of view of evaluation of various policy options and the assessment of the impact of land usage change, on natural and socio-economic conditions. The GB Model determined urban densities numerically, analyzed the resulting density patterns and compared them with those derived from the empirical analyses of actual urban data. In view of these models it is purposed the adoption of new spatial structures, and for the existing urban areas possible solutions are suggested. Some urban density patterns were simulated by varying the model parameters. The outcome of these simulations are analyzed and the Distance-Density patterns are studied using Pivotal technique of FGDM. Furthermore, the GB model was constructed to study the effect of increasing the size of the city and decreasing the distance friction coefficient  $\beta$  on population distribution. A marked flattening of distance-density curves is evident in all the cases.



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